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SHAPE OF POROUS COOLED REGION FOR SURFACE HEAT FLUX AND TEMPERATURE BOTH SPECIFIED

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NOMENCLATURE*

C_p, specific heat at constant pressure;
G, mass flow rate of coolant per unit length normal to x, y plane;
k_m, effective thermal conductivity of porous region;
l, position along boundary S;
M for incompressible flow

$$M = \frac{1}{2}\frac{\mu}{\mu_\infty}\frac{t_\infty}{t};$$

for compressible flow $M = \mu/\mu_\infty$;
P, for incompressible flow $P = p/p_\infty$; for compressible flow $P = (p/p_\infty)^2$;
p, pressure;
Q_{total}, heat conducted into surface S per unit length normal to x, y plane;
T, temperature ratio t/t_∞;
t, absolute temperature;
$\boldsymbol{u}$, velocity vector;
X, Y, dimensionless coordinates $x/h_r, y/h_r$, where h_r is defined in equation (8);
α, surface absorptivity for incident radiation;
κ, permeability of porous material;
λ, parameter $\rho_\infty C_p \kappa p_\infty / 2\mu_\infty k_m$;
μ, fluid viscosity;
$\tilde{\nabla}$, dimensionless gradient in X, Y plane, $\hat{\imath}(\partial/\partial X) + \hat{\jmath}(\partial/\partial Y)$.

Subscripts
i, insulated and impervious surface;
r, reference condition;
s, on surface where coolant exits from porous medium;
0, on surface where coolant enters porous medium.

* Other symbols are defined in text.

INTRODUCTION

AN EFFECTIVE cooling technique can be obtained by utilizing porous materials so that coolant can be forced out through the surface that is subjected to a heat load. For a given pumping pressure the amount of coolant passing through a location on the cooled surface will depend on the flow resistance along the path through the porous region to that location. The shape of the porous region will regulate the flow resistance and hence the local cooling capability along the surface. If a heat flux distribution is imposed along the surface and it is also desired to maintain the surface at a uniform design temperature, it is required to know what shape the porous region should have to meet these conditions.

In [1] a heat transfer analysis was devised for a three-dimensional porous cooled region of specified shape. The analysis obtained the region temperature distribution in terms of a potential function found by solving Laplace's equation in the geometry of the porous region subject to simple boundary conditions. Since steady-state heat conduction is governed by Laplace's equation, that analysis related the porous cooled problem to heat conduction theory. In [2] a heat conduction analysis was developed to determine the shape of a conducting nonporous region with a surface having simultaneously a uniform temperature and a specified heat flux distribution. The purpose of this note is to show how the results of [1] and [2] can be combined to determine the required shape of a porous cooled region. Since this note is rather brief, it would help for the reader to become familiar with [1] and [2].

ANALYSIS

The general two-dimensional configuration considered here is shown in Fig. 1. A coolant reservoir at p_∞, t_∞ is adjacent to S_0 which is open to flow. Coolant exits through S which receives a heat flux distribution $q_s(l)$. The remaining boundary S_i is impervious to flow and is insulated. As the coolant approaches the inlet surface S_0 the pressure drop from flow acceleration is small compared with the drop through the porous material so that $p_0 \approx p_\infty$. Since p_0 and p_s are constants, the velocities at S_0 and S are locally normal to these surfaces.

The governing equations and boundary conditions are:

Conservation of mass

$$\nabla . (\rho \boldsymbol{u}) = 0 \qquad \text{(Compressible)} \tag{1a}$$

$$\nabla . \boldsymbol{u} = 0 \qquad \text{(Incompressible)} \tag{1b}$$

Darcy's law

$$\boldsymbol{u} = -\frac{\kappa}{\mu(t)} \nabla p \tag{2}$$

Energy conservation

$$\nabla . \boldsymbol{q} = 0 \quad \text{where } \boldsymbol{q} = -k_m \nabla t + \rho \boldsymbol{u} C_p t \tag{3}$$

Perfect gas law (for compressible case)

$$p = \rho R t \tag{4}$$

Boundary conditic 's:

$$\left.\begin{aligned} k_m \hat{n}_0 . \nabla t &= \rho C_p (t - t_\infty) \hat{n}_0 . \boldsymbol{u} \quad (5a) \\ p = p_0 &= p_\infty = \text{constant} \quad (5b) \end{aligned}\right\} x, y \quad \text{on} \quad S_0$$

$$\left.\begin{aligned} t &= t_s = \text{constant} \quad (6a) \\ k_m \hat{n}_s . \nabla t &= q_s(l) \quad (6b) \\ p &= p_s = \text{constant} \quad (6c) \end{aligned}\right\} x, y \quad \text{on} \quad S$$

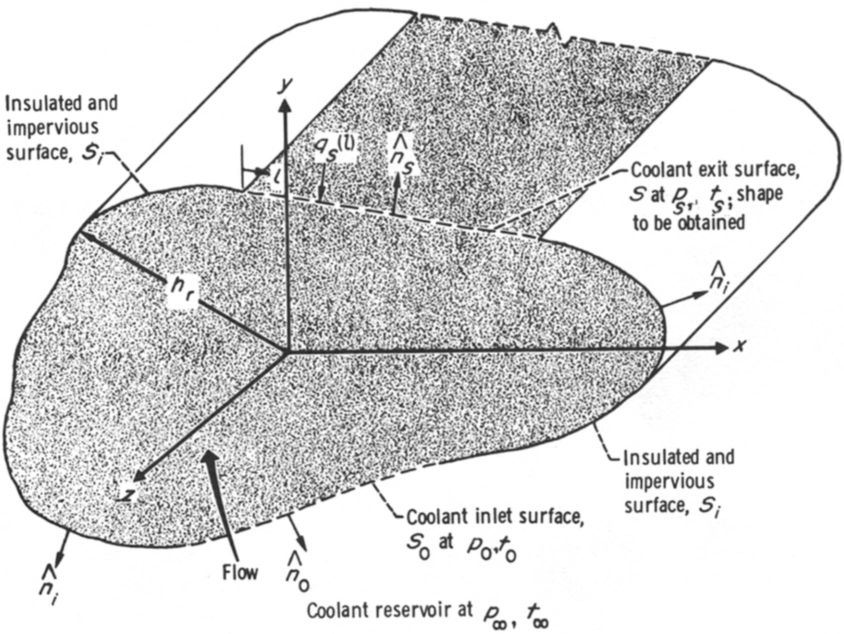

FIG. 1. Two-dimensional porous region with unknown shape of coolant exit surface, $p_\infty > p_s, t_s > t_\infty$.

$$\left.\begin{aligned}\hat{n}_i.\boldsymbol{u} &= 0\\ \hat{n}_i.\boldsymbol{q} &= 0\end{aligned}\right\} x, y \quad \text{on} \quad S_i. \qquad \begin{matrix}(7a)\\(7b)\end{matrix}$$

These equations are taken from [1] with the exception of equation (6b) which accounts for the specified heat flux along S. The solution in [1] satisfies all conditions except equation (6b) which can be satisfied here by letting S have an unspecified shape to be determined.

In [1] all lengths are non-dimensionalized by an arbitrary length h_r. For the present situation let

$$h_r = k_m(t_s - t_\infty)|\Phi_s|/q_{s,r} \qquad (8)$$

where $q_{s,r}$ is a reference heat flux, and Φ_s is given later. The analysis in [1] with the addition of equation (6b) in dimensionless form, shows that the present solution can then be obtained in terms of a potential function ϕ found from

$$\tilde{\nabla}^2\phi = 0 \quad X, Y \text{ in porous region} \qquad (9)$$

with the boundary conditions

$$\phi = 0 \quad X, Y \text{ on } S_0 \qquad (10)$$

$$\left.\begin{aligned}\phi &= 1\\ \hat{n}_s.\tilde{\nabla}\phi &= q_s(l)/q_{s,r}\end{aligned}\right\} X, Y \text{ on } S \qquad \begin{matrix}(11a)\\(11b)\end{matrix}$$

$$\hat{n}_i.\tilde{\nabla}\phi = 0 \quad X, Y \text{ on } S_i. \qquad (12)$$

The temperature distribution in the porous region is expressed in terms of $\phi(X, Y)$ as

$$T = 1 + (T_s - 1)\exp[\Phi_s(1 - \phi)] \qquad (13)$$

where

$$\Phi_s = -\ln\left(\frac{T_s - 1}{T_0 - 1}\right) \qquad (14)$$

and T_0 is found from the pressure ratio by using

$$P_s - 1 = \frac{1}{\lambda}\int_{T_0}^{T_s} \frac{MT}{1 - T}\,\mathrm{d}T. \qquad (15)$$

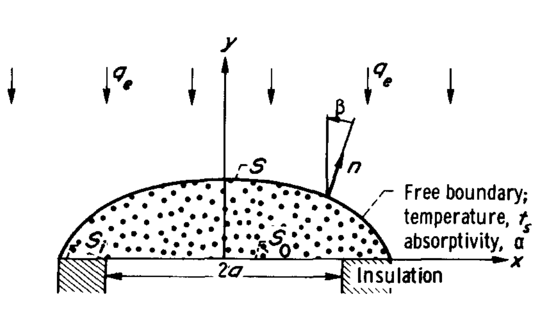

FIG. 2. Cross section of two-dimensional porous region with free boundary at uniform temperature and with uniform absorptivity exposed to unidirectional radiation.

Although the temperature distribution is of some interest, the main result to be obtained here is the shape of the boundary S. This will provide the proper coolant flow distribution to remove the specified surface heat flux while maintaining the desired surface temperature. For this part of the solution, the technique and results of [2] can be directly applied as the governing equation and boundary conditions in [2] are the same as the present equations (9)–(12).

The example treated in [2] is a nonporous region with isothermal surface S of unknown shape being subjected to a unidirectional radiative heat flux q_e, Fig. 2. The local heat absorption along S is $q_s(l) = \alpha q_e \cos\beta$. Using the boundary condition (11b) with $q_{s,r} = \alpha q_e$, and resolving $q_s(l)$ into X and Y components gives the derivative conditions for ϕ in Fig. 3.

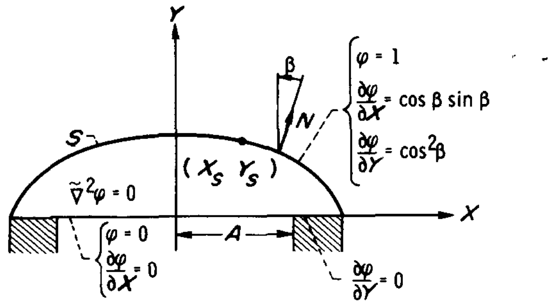

FIG. 3. Porous region and boundary conditions in dimensionless physical plane.

The solution of the boundary value problem in Fig. 3 for the shape of surface S is obtained in [2] by conformal mapping, with the resulting coordinates along S being

$$\frac{X_s}{A} = \frac{x_s}{a} = 1 + \frac{2}{\pi A K(\sqrt{(1 - b^2)})} \times \left\{\int_b^1 \ln\left(\frac{b\sqrt{(1-\delta^2)} + \delta\sqrt{(1-b^2)}}{\sqrt{(\delta^2 - b^2)}}\right)\frac{\mathrm{d}\delta}{\sqrt{(\delta^2 - b^2)}\sqrt{(1-\delta^2)}} - \frac{\pi}{2}\left[F\left(\sin^{-1}\frac{\xi}{b}, b\right) + K(b)\right]\right\} \qquad (16a)$$

where $-b < \xi \leqslant 0$

$$\frac{Y_s}{A} = \frac{y_s}{a} = \frac{2}{\pi A K(\sqrt{(1 - b^2)})} \times \int_{-b}^{\xi} \ln\left(\frac{b\sqrt{(1-\xi^2)} - \xi\sqrt{(1-b^2)}}{\sqrt{(b^2-\xi^2)}}\right)\frac{\mathrm{d}\xi}{\sqrt{(b^2-\xi^2)}\sqrt{(1-\xi^2)}} \qquad (16b)$$

where $-b < \xi \leqslant 0$

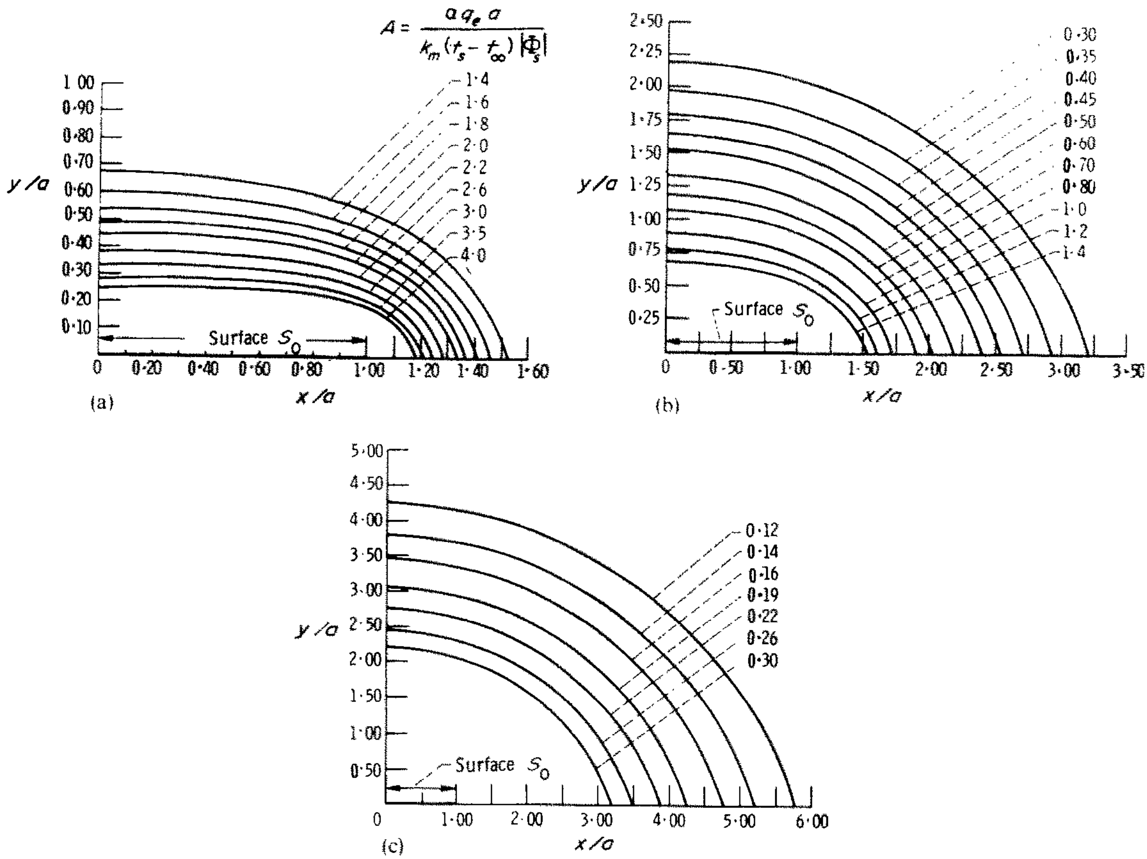

FIG. 4. Shape of porous region for various values of the physical parameter. (a) Thin regions. (b) Regions of intermediate thickness. (c) Thick regions.

The ξ is a dummy variable of integration, F is the elliptic integral of the first kind, and K is the complete elliptic integral of the first kind, [3]. The physical parameter A is the dimensionless half-width of surface S_0 shown in Fig. 3, and for the porous problem is equal to

$$A = \frac{a}{h_r} = \alpha q_e a / k_m (t_s - t_\infty) |\Phi_s|. \tag{17}$$

From [2] the quantity b is related to A by

$$A = \frac{2}{\pi K(\sqrt{(1-b^2)})} \times \int_0^1 \tan^{-1}\left(\frac{b\sqrt{(1-v^2)}}{\sqrt{(1-b^2)}}\right) \frac{dv}{\sqrt{[(1-v^2b^2)(1-v^2)]}}. \tag{18}$$

Figure 4 (from [2]) shows the resulting shapes of the surface S for various values of A.

The heat conducted into the porous region depends on the normal temperature derivative at S. The only change from [2] to the present case is the use of h_r from equation (8) to non-dimensionalize the normal direction rather than the scale factor γ of [2]. Making this change yields Fig. 5 for the heat flow into S. An overall energy balance gives

$$Q_{\text{total}} = C_p G(t_s - t_\infty)$$

so that Fig. 5 also provides the total coolant flow rate.

CONCLUSIONS

An analogy has been shown between heat transfer in a three-dimensional porous cooled region and ordinary heat conduction. A heat conduction analysis in a nonporous

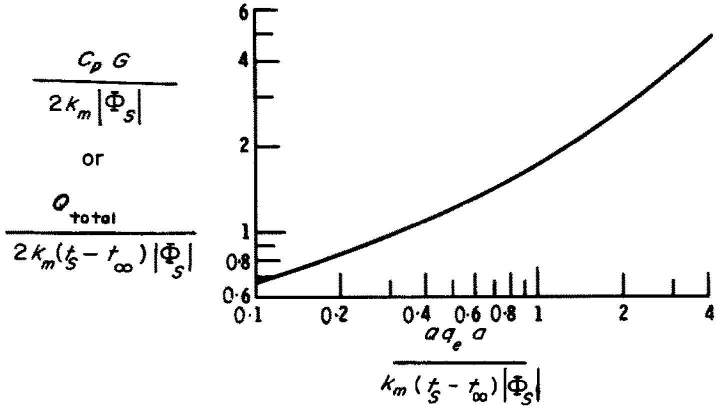

FIG. 5. Dimensionless coolant or heat flow as a function of physical parameter involving absorbed incident radiation.

region having a free boundary can then be utilized to obtain results for a related porous cooled problem. In this way a porous region shape is obtained that will provide proper cooling for a specified heat flux variation along a surface while maintaining a specified uniform surface temperature. A two-dimensional example is given where a surface is subjected to thermal radiation from one direction. The analysis applies for three dimensions, but the example is limited to two dimensions because conformal mapping is used to obtain the free boundary shape.

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A GENERAL EXPRESSION FOR THE RATE OF EVAPORATION OF A LAYER OF LIQUID ON A SOLID BODY

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NOMENCLATURE

C, solution of $C\,e^{C^2}\operatorname{erfc} C = 1/(\lambda\sqrt{\pi})$;
C_1, constant in $R = C_1 t^n$;
C_2, constant in $\delta_0 = C_2\sqrt{(\nu t_g)}$;
c_p, specific heat capacity;
$F(\lambda, \phi, \beta^2, \gamma, t^+)$, function defined in text as $(1 - \delta^+)$;
h_{fg}, latent heat of vaporization;
h_i, surface heat transfer coefficient at interface between liquid and vapour;